Prediction of Dynamic Properties of Plastic Foams from Constant-Strain Rate Measurements

ORI RAMON and JOSEPH MILTZ,* *Packaging Laboratory, Department of Food Engineering and Biotechnology Technion, Israel Institute of Technology, Haifa, 32000 Israel*

Synopsis

This study demonstrated how cushioning curves can be predicted from constant-strain rate measurements and a limited number of stress-relaxation or drop experiments. For rate-independent foams, only one stress-strain curve and the proposed dynamic model are required. For rate-dependent foams, however, a computer and appropriate software are also needed to predict accurately cushioning curves.

INTRODUCTION

Fragile products need protection against shock and vibration encountered during their shipment. The level of protection depends on the fragility of the packaged item and on the severity of the environment (transit conditions). The required protection is normally achieved by cushioning materials, primarily plastic foams. These foams damp applied forces and absorb energy during their compression. Cushion design is based on cushioning curves that describe the G-level (maximum dimensionless deceleration, to be defined later) that a foam transmits to the packaged item as a function of the static stress applied on the foam.[1,2] Different curves are, however, obtained for different foam thicknesses, foam densities, cell sizes and shapes, and drop heights. Consequently, characterization of the cushioning properties of even one foam requires a tremendous amount of experimental work to determine cushioning curves as a function of the above-mentioned parameters. As a result, only a limited number of such curves, for specific foams, for several foam thicknesses and drop heights have been described in the literature.

The present study extends our previous work by describing a more advanced approach for the more accurate predictions of cushioning curves.

Theoretical Background

A mechanical shock encountered by a dropped product during its shipment is a complex transient motion involving a combination of decaying vibrations at different frequencies. It is well known, however, that the first impact with the ground is the most severe one. When a cushioning material is used to

* To whom correspondence should be addressed.

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protect a fragile product, the equation of motion during the shock can be described by the following equation:

$$m\ddot{y} = mg - F(y, \dot{y}), \tag{1}$$

where m is the mass of the product, g is the gravity, and y, $\dot{y}$, and $\ddot{y}$ are the displacement, rate of displacement (velocity), and deceleration of the packaged product, respectively.

When a cushion is being used to protect the item, eq. (1) can be represented in the form:

$$\frac{m}{A}\ddot{y} = \frac{m}{A}g - \sigma(\epsilon, \dot{\epsilon}), \tag{2}$$

where A is the area of the foam in contact with the product and σ is the stress. The strain, ϵ, and strain rate, $\dot{\epsilon}$, are the ratios of the displacement, y, and velocity, $\dot{y}$, and the foam thickness, h, namely:

$$\epsilon = y(t)/h \tag{3}$$

$$\dot{\epsilon} = \dot{y}(t)/h. \tag{4}$$

When the coordinates are chosen so that the positive direction of y is downward and $y = 0$ at the top of the undeformed foam while touching the ground, then the boundary conditions for eq. (1) are

$$\dot{y}_{t=0} = 2gH \tag{5}$$

$$\ddot{y}_{t=0} = g, \tag{6}$$

where H is the drop height of the packaged item. If the mechanical (stress-strain) behavior of the foam is known, eqs. (1) and (2) could be solved analytically or numerically (depending on the function F (or σ)) to obtain the displacement, velocity, and deceleration of the mass, m, as a function of time.

In the past, several attempts to describe the mechanical behavior, in compression, of plastic foams were made by Rusch,[3,4] Hilyard,[5] Cost et al.,[6] Cousins,[7] Yossifon and Szanto,[8] Shuttleworth et al.,[9] and Meinecke and Clark.[10] Miltz and Gruenbaum[11,12] tried to predict cushioning curves from compressive measurements at low strain rates. They have also proposed two parameters, the "Efficiency" and "Ideality," which enable one to select the most appropriate foam candidates for a specific application. These predictions were based, however, on measurements carried out at much lower strain rates than encountered during a shock and on a rather simple model. As a result, discrepancies of 30%–50% between the calculated and measured curves were observed. Recently, Miltz et al.[13-15] have described two more accurate models to predict stress-strain curves, at any strain rate, from one stress-strain curve and a limited number of stress-relaxation curves.

A second approach used to look at the protective behavior of plastic foams under impact conditions was based on energy considerations as outlined by

Mindlin,[16] Meinecke and Clark,[10] and Miltz and Gruenbaum.[11,12] When a packaged item is dropped from a height H, the potential energy is transferred into kinetic energy and, on impacting the ground, into energy absorbed by the cushion. Thus, at the maximum deflection of the foam, y_m, the following equation applies:

$$mg(H + y_m) = \int_0^{y_m} F(y, \dot{y})\, dy. \tag{7}$$

Normally, y_m is negligible compared to H, and therefore eq. (7) can be rearranged to the form

$$mg = \frac{1}{H} \int_0^{y_m} F(y, \dot{y})\, dy. \tag{8}$$

If the maximum dimensionless deceleration (the ratio between the actual deceleration and that of gravity) that the protected body can withstand is designated as G_m, then:

$$G_m = \frac{m\ddot{y}_m}{mg} = \frac{F_m}{mg}, \tag{9}$$

where F_m is the maximum force applied on the protected item. This force is normally, but not always (as will be explained below), equal to $F(y_m)$. Combining eqs. (8) and (9) results in

$$G_m = \frac{F_m H}{\int_0^{y_m} F(y, \dot{y})\, dy} = \frac{F_m H}{E_s}, \tag{10}$$

where

$$E_s = \int_0^{y_m} F(y, \dot{y})\, dy. \tag{11}$$

The static stress, σ_{st}, normally used to describe cushioning curves, is defined as

$$\sigma_{st} = \frac{mg}{A}. \tag{12}$$

Combining eqs. (7), (11), and (12) results in

$$\sigma_{st} = \frac{E_s}{HA}. \tag{13}$$

MATERIALS AND METHODS

Materials

The materials used were a variety of polymeric foams that are used as cushioning materials. These included three chip-bonded semiflexible polyurethane (PUR) foams of densities 0.11, 0.14, and 0.24 g/cc, closed-cell expanded polystyrene (PS), 0.032 g/cc in density, and a crosslinked polyethylene (PE) foam, 0.041 g/cc in density. Several thicknesses of the different foams were studied. Round samples of 100 cm^2 in area were used. In Table I, the dimensions and some properties of these foams are summarized.

Methods

The constant rate compressive measurements were carried out on J & J Lloyds Type 5002 and MTS-810 Universal testers. The speed of testing was in the range of 0.1–300 cm/min.

The dynamic compressive tests were conducted as free-fall drops of different masses (containing an accelerometer) on the studied foams in a special instrument designed for this purpose and shown in Figure 1. The falling mass was released from a preset height, and when passing very close to the top of the foam, it activated a trigger that started the acquisition of data by a Digital LSI 11/23 minicomputer. The analog data were converted to digital values using sophisticated, high-accuracy, 12 bit, A/D boards connected to a real-time clock that enabled the establishment of a very accurate time-base recording menu. The velocity and displacement as a function of time were determined by consecutive integration of the acceleration pulse measured by the accelerometer.

An electrooptical meter type VS-300 DT by MTS was used to measure the exact velocity before impact (resulting from the free-fall drop).

RESULTS AND DISCUSSION

Figure 2(a) is a typical pulse generated by the computer as measured by the accelerometer for the variables shown in the figure. First integration of this pulse (acceleration) with respect of time provided the velocity, and a second integration provided the displacement as a function of time as shown in Figures

TABLE I
Some Characteristic Parameters of the Investigated Foams

Foam	Density (g/cc)	Average cell size (mm)	Modulus of elasticity (kg/cm^2)
PUWP	0.110	0.250	1.00
PUBP	0.135	0.400	1.16
PUOP	0.235	0.300	3.55
PE	0.040	0.500	4.79
PS	0.032	0.075	23.0

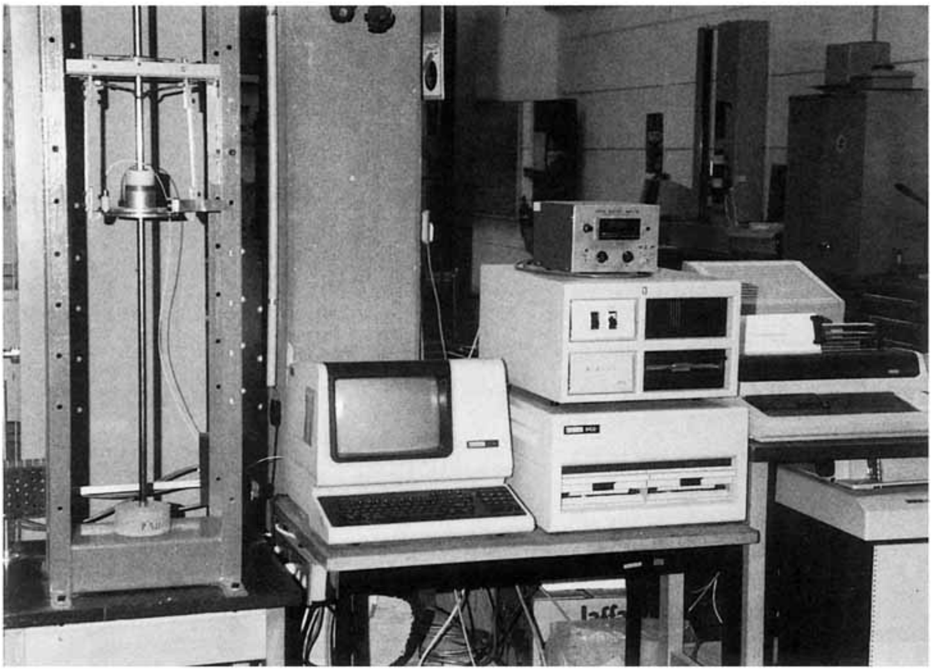

Fig. 1. Drop instrument and digital LSI 11/23 computer.

2(b) and (c). Because of the viscoelastic behavior of the foams, the maximum displacement does not necessarily occur at the same time as does the maximum acceleration, as is evident from Figure 2. Since the dropped mass and foam area were known, the dynamic force-deformation (or stress-strain) curves could be constructed with the aid of a special computer program designed for this purpose.

In previous publications we have shown[13,14] that the relation among stress, strain, and strain rate for polymeric foams used as cushioning materials can be described by the following equation:

$$\sigma = K_r f(\epsilon) \dot{\epsilon}^n \frac{1}{1-n} \epsilon^{1-n}, \qquad (14)$$

where σ is the stress ϵ and $\dot{\epsilon}$ are the strain and strain rates, respectively, and K_r, $f(\epsilon)$, and n are parameters obtained from a limited number of stress-relaxation experiments for each of the foams. K_r is a reference modulus, whereas $f(\epsilon)$ is a strain function that depends on the strain only. The parameter n can be either a constant or a strain and strain-rate dependent, depending on the kind and nature of the foam. We have proposed that Eq. (14) is a constitutive equation, even though it was based on results for constant, relatively slow, strain rates. However, if this equation is really a constitutive one, then it should apply for any strain rate, including dynamic measurements. For the latter, the measured force could be described by

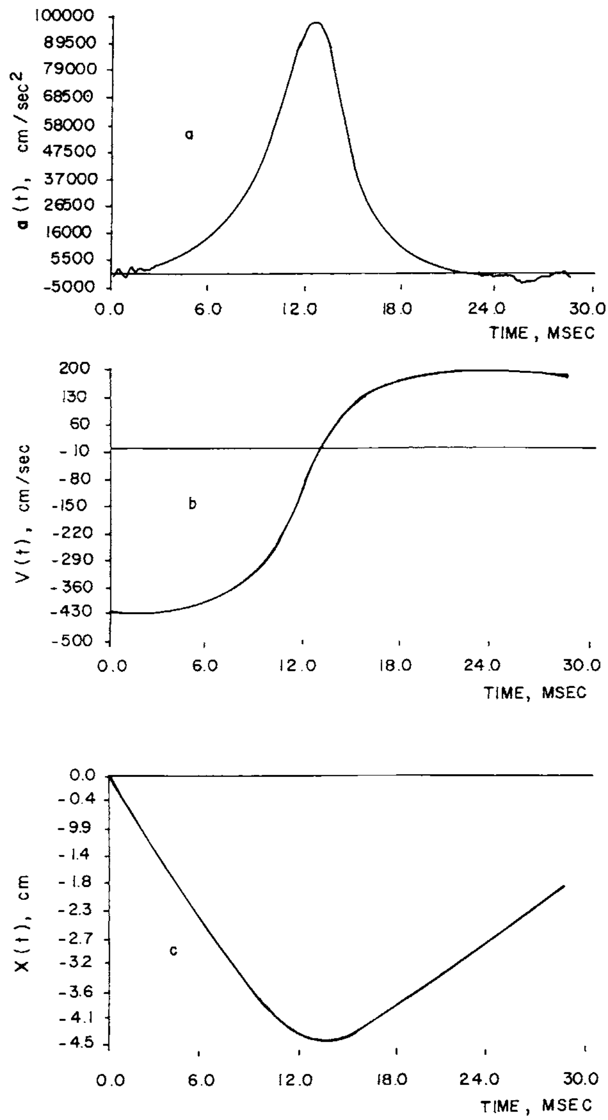

Fig. 2. (a, b, c) The change of acceleration, velocity, and displacement (of the dropped mass) with time.

$$F_{dyn} = AK_r f(\epsilon_{dyn}) \frac{1}{1-n} (\dot{\epsilon}_{dyn})^n (\epsilon_{dyn})^{1-n}, \tag{15}$$

where A is the cross-sectional area of the foam. The strain and strain rate in eq. (15) were calculated by the computer program from eqs. (3) and (4).

In Figure 3, the calculated curve from the measured acceleration peak of the PUBP foam is compared to the one calculated using eq. (15). It is evident that the agreement between the curves is excellent. Similar curves were obtained for the other PUR foams for which n is rate independent.

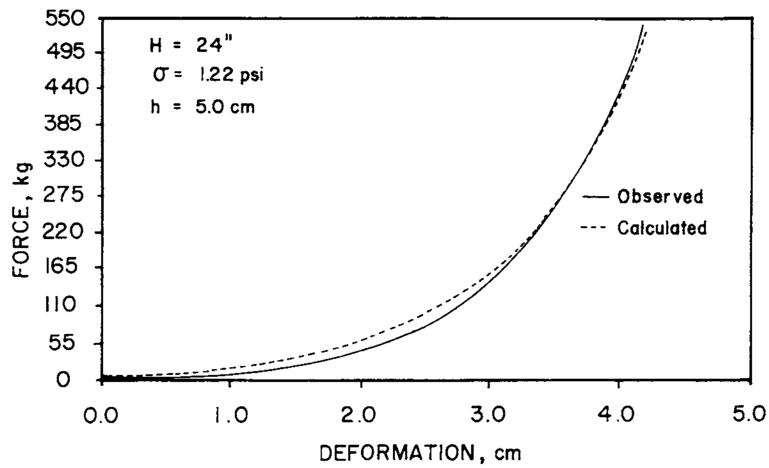

Fig. 3. Calculated and measured force-deformation curve for PUBP foam.

In the case of the closed-cell polyethylene foams for which n is strain and strain-rate dependent, the computer program used $n(\epsilon, \dot{\epsilon})$ instead of n in eq. (15) to determine the dynamic-force deformation curve. In Figure 4, the calculated curves from the acceleration peak and that from eq. (15) are compared for the PE foam. Although the agreement is slightly poorer than for the PUR foam, a very good agreement between the two curves still exists.

It was pointed out in the Introduction that cushioning curves are normally used for cushion design. Such curves can be constructed from the measured parameters and eqs. (10) and (12) for any drop height, any mass (any static stress), and any foam thickness. On the other hand, experimentally determined cushioning curves exist (or can be obtained) for a specific and limited number of drop heights and foam thickness because of the tremendous amount of experimental work involved in the construction of such curves.

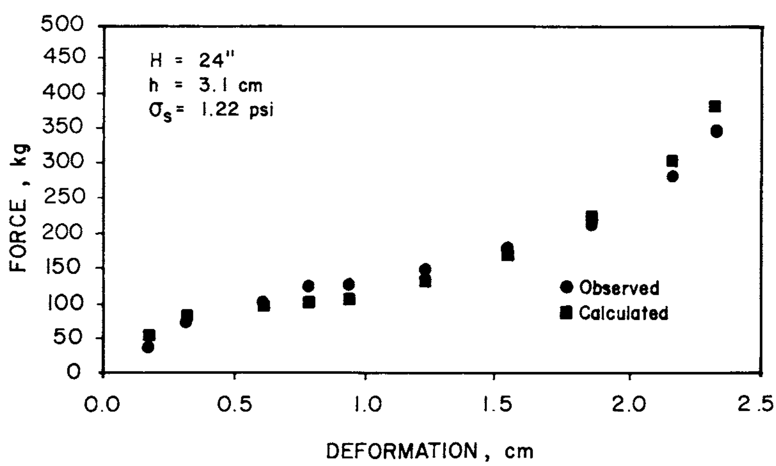

Fig. 4. Calculated and measured force-deformation curve for PE foam.

In previous publications[13,14] we have shown how force-deformation curves for plastic foams could be predicted for any strain rate based on a limited number of stress-relaxation curves and one stress-strain curve and using the "Reference Model" (see Refs. 13 and 14) or the "Modified Boltzman Model" [eqs. (14) and (15)].

For rate-independent foams, like expanded PS, one can accurately predict cushioning curves using the dynamic model [eqs. (10) and (13)], but E_S can be evaluated from slow, constant slow-rate experiments. The use of the "Reference Model" or the "Modified Boltzman Model" are not required in this case.

A comparison between one so calculated and an experimentally determined cushioning curve is shown in Figure 5. It can be seen that the agreement between the two curves is very good. Similar curves were obtained for drop heights and foam thicknesses. For the rate-dependent PUR and PE foams for which the parameter n changes also with the strain rate, a different approach was applied. Most constant-rate tensile or compression testers do not run at such high speeds as encountered during drop or shock experiments. Thus, it is difficult to predict the value of n at these high speeds from stress-relaxation experiments. On the other hand, the predicted cushioning curves depend very much on the value of n used in the computer program, as can be seen in Figure 6. Meinecke et al.[17] showed that the initial modulus of a foam, E_0, changes with the strain rate in the following way:

$$E_0(\dot{\epsilon}) = K_1\dot{\epsilon}^a, \tag{16}$$

where K_1 is the modulus at some reference strain rate and a is actually the same parameter as n used in the present study. It was also shown that a is a constant throughout a limited range of the strain rate and increases to a higher constant level at higher strain rates.

Hong et al.[18] have shown that the relaxation modulus changes with temperature in a similar way as eq. (16) where the temperature replaces the strain

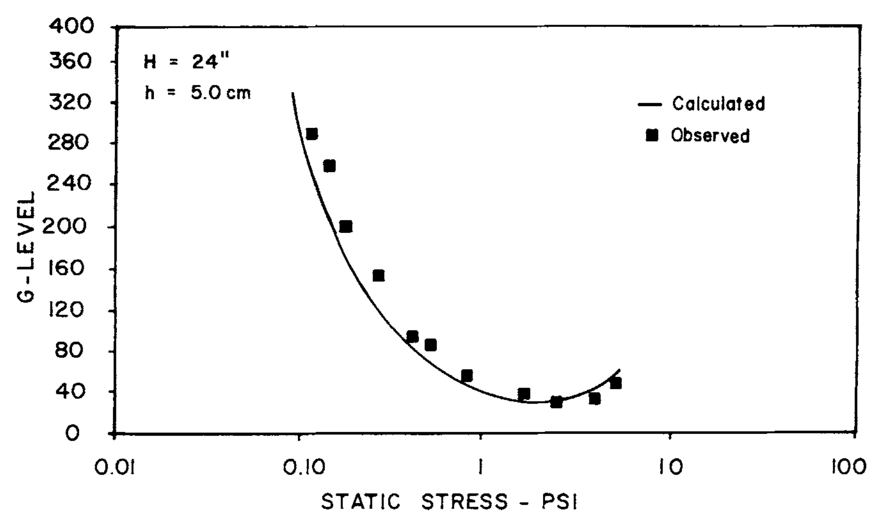

Fig. 5. Predicted and measured cushioning curve for PS foam.

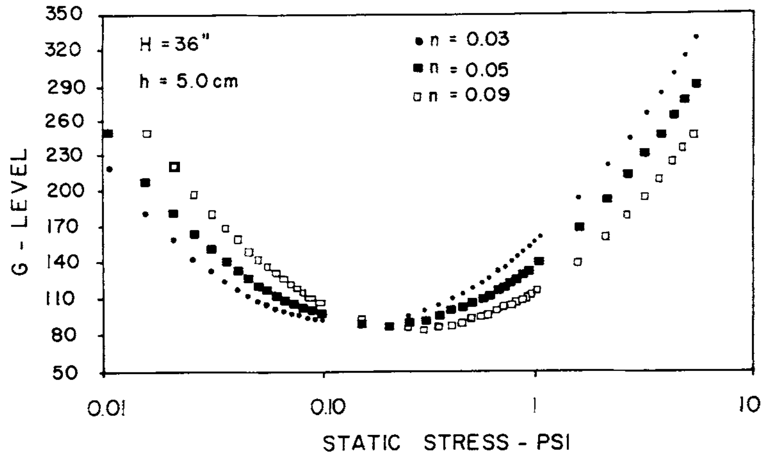

Fig. 6. The effect of parameter n on cushioning curves of PUBP foam.

rate and the power, a, is negative. They have also found that a is constant in a limited range of temperatures and increases to a higher constant level at lower temperatures. From the viscoelastic theory, these two effects are similar.

Using the method of data acquisition (with an accelerometer, a computer, and adequate software) during the free-fall drop from different heights enabled us to calculate the variation of the initial modulus with strain rate (namely, parameter n) at rates encountered during a drop (or shock) and thus to predict more accurately cushioning curves. In Figures 7 and 8, the calculated cushioning curves for a PUR and PE foams are compared to the experimentally determined ones for one condition. It can be seen that the agreement is very good. Similar curves were obtained for different conditions (different drop heights and foam thicknesses).

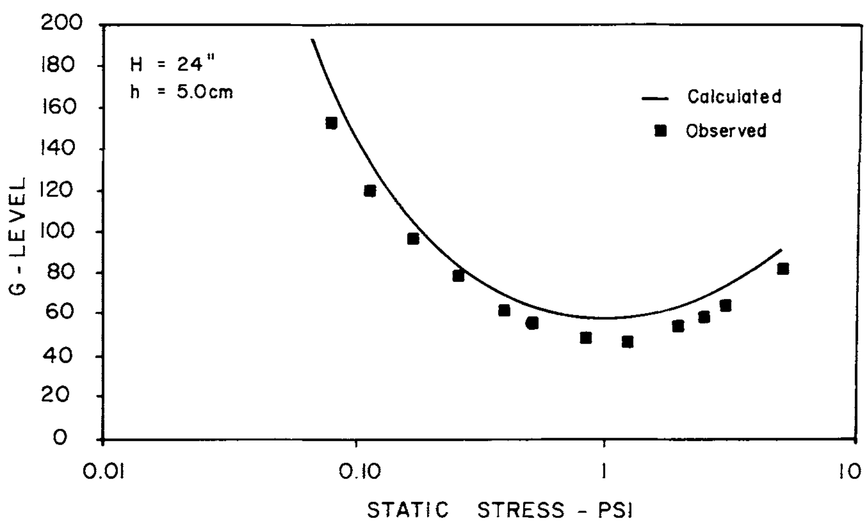

Fig. 7. Predicted and measured cushioning curve for PUOP foam.

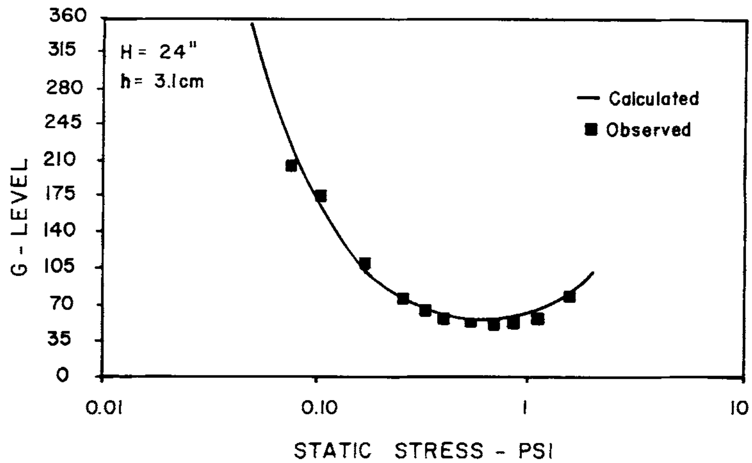

Fig. 8. Predicted and measured cushioning curve for PE foam.

To summarize, it was demonstrated that for rate-independent foams cushioning curves can be predicted accurately by using one stress-strain curve and eqs. (10) and (13) developed for the dynamic model. For rate-dependent foams, a more complicated procedure requiring also a limited number of stress-relaxation experiments and involving a computer and appropriate software are required for such predictions. But by doing so, quite accurate force-deformation and cushioning curves can be obtained.

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